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**Nonlinear Mixed-Effects Height-Diameter Model
for the Management of Nasarawa State
University *Tectona grandis* Plantation, Lafia**

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NONLINEAR MIXED-EFFECTS HEIGHT-DIAMETER MODEL FOR THE MANAGEMENT OF NASARAWA STATE UNIVERSITY *TECTONA GRANDIS* PLANTATION, LAFIA

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ABSTRACT: The study assessed nonlinear mixed-effects height-diameter models with a view to developing a height diameter model, applying a nonlinear mixed effects height-diameter modeling method on asymptotic and Michaelis Menten models and examining the fixed and random effects estimates of the models for the management of Nasarawa State University Forestry *Tectona grandis* plantation. Thirty (30) temporary sample plots of 0.01 ha size were randomly selected from the sampling frame with 30% sampling intensity. Diameter at breast height (DBH) and total heights of the trees within each selected plot were measured. The result of nonlinear height diameter model revealed that model two (2) had the lowest AIC (-1168.253), BIC (-1157.469) and RSE (0.02738) of model selection indices when compared to other three (3) models applied. Model (2) was the best for predicting tree heights in the study area. The result of nonlinear mixed effect height diameter model showed that random effects produced a better line of best fit when compared with fixed effects line of best fit of the asymptotic and Michaelis Menten models. In addition, it indicated an increasing trend of deterministic line with minimal residuals better than fixed effects line of best fit. The Standardized residuals was plotted against the fitted values of asymptotic and Michaelis Menten models in order to test for the normality of the data set from the plantation. The result further showed that the *Tectona grandis* data set was evenly distributed around the deterministic line. Therefore, nonlinear mixed-effects height diameter model provides access to both fixed effects and random effect estimates of data set in which random effects estimate provided a better estimates for tree heights unlike the commonly used generalized ordinary least square height-diameter modeling method.

Keywords: Nonlinear height diameter model, mixed effects, random effects, *Tectona grandis*

1. INTRODUCTION

Tectona grandis is a valuable tree species, particularly known for its artistic wood grains and durability which attracts high demand in Lafia community, Nasarawa State, Nigeria. The wood is used for boat and ship building, in addition to construction, decorative veneers, furniture, cabinets, musical instruments, woodcarving and floors (Espitia *et al.*, 2011). Measurements of stem diameter and wood density can be obtained easily, tree

height measurements in forests plantation can be time consuming and difficult, as the top of some emergent trees cannot be clearly viewed. However, studies have shown that incorporating tree diameter can significantly improve tree height estimates, because significant variations exist in tree height for a given stem diameter in tropical forests (Vieilledent *et al.*, 2012; Banin *et al.*, 2012; Rutishauser *et al.*, 2013). Therefore, there is a need to explore different modeling methods to quantify tree heights by incorporating diameter at breast heights.

Models are mathematical expressions which relate tree yield to trees measurable variables such as stem diameter, height, volume among others to estimate average content of standing trees of various sizes and species (Avery and Burkhart, 2002). Studies have encouraged the use of non-destructive methods of tree yield estimation such as allometric models (Adeyemi and Moshood, 2019). Tree height and diameter measurements are basic input information to various forest models that serve as important tools in forest management. Various forest models such as volume, site index, growth and yield, biomass, and carbon budget models are based on the measurement of tree heights. For any sample inventory design such as national forest inventory sample plots, permanent sample plots, and temporary sample plots, diameter are measured for all tree, but only few selected trees are measured for total heights when applying tree height diameter modeling. Measuring the heights of a standing tree is much more difficult, time consuming and costlier than measuring its diameters. In such a situation, a sample plot-specific height-diameter model is necessary for accurate prediction of the missing height measurements of the trees for which diameters are measured.

The height diameter relationship differs from one stand to another due to difference in site quality, stand age, silvicultural treatments, and even within the same stand due to a differing competitive situation among the trees (Pretzsch, 2009). Therefore, height diameter models need to be made stand-specific in order to increase the prediction accuracy. A single height-diameter model applicable to a large forest area can be developed using one of the two parametric approaches. The first approach uses site quality measure (site index or dominant height) and stand density measure (basal area, stem number, mean diameter) as additional covariates in the height-diameter models (Castedo-Dorado *et al.*, 2006). These models are commonly known as the generalized ordinary least square height-diameter models. The second approach uses site quality and stand density measures as additional covariates and subject-specific effects as random effects in the height-diameter models (Mehtatalo *et al.*, 2015). These models are often termed as the generalized effect models (hereafter termed as mixed effect model) and are more accurate than the generalized ordinary least square height-diameter models (Crecente-Campo *et al.*, 2010). The measurement of diameter at breast height and tree total height is fundamental to both developing and applying height-diameter models.

Tree models are vital tools for forest plantation management. One of the greatest challenges in model development is measuring tree variables and data gathering. In forest measurement, diameter at breast

height of a tree could be measured easily and accurately, but the measurement of total tree height is relatively complex, time consuming and expensive. Furthermore, some site conditions and tree composition especially in tropical forests may prevent accurate height measurements of all measured diameter at breast height trees as it may not be possible to unambiguously observe an appropriate vantage point. Also, the estimating power of nonlinear mixed-effects height-diameter modeling method is rarely known unlike the generalized ordinary least square height-diameter modeling method. Therefore, the study is restricted to non-linear mixed-effect height-diameter model for the management of Nasarawa State University *Tectona grandis* plantation, Lafia with specific interest in developing a height diameter model, applying a non-linear mixed effect height-diameter modeling method and providing fixed and random effects estimates of asymptotic and Michaelis Menten models for the management of the plantation.

2. MATERIALS AND METHODS

2.1. Study Area

The study was carried out at Nasarawa State University Forestry *Tectona gradis* Plantation, Faculty of Agriculture, Shabu-Lafia Campus. The plantation is located between longitude 8°33'N and latitude 8°34'E, in guinea savannah zone of North Central Nigeria at an altitude of about 177 m above the sea level. The mean maximum and minimum temperature range between 35.06 °C to 36.40 °C and 20.16 °C to 20.25 °C while the mean monthly relative humidity and rainfalls are 74.67% and 168.90 mm respectively. Tropical ferruginous soils make up the major soil units found in Lafia Local Government Area. The parent material for the soils are from basement complex and sedimentary formations in the area. Laterite crust occurs extensively on the basement complex rocks while hydromorphic soils are common along river Benue trough and flood plains of major rivers (Lyam, 2000).

2.2. Sampling technique

Thirty (30) temporary sample plots of 0.01 ha size was randomly selected from the sampling frame of with 30% sampling intensity. The plots were randomly selected and individual trees within each plot were measured. The diameter at breast height (at 1.3 m above ground level) was measured using a diameter tape. Total heights was measured using Haga altimeter and Global Positioning System (GPS) was used to take the coordinates of the study area.

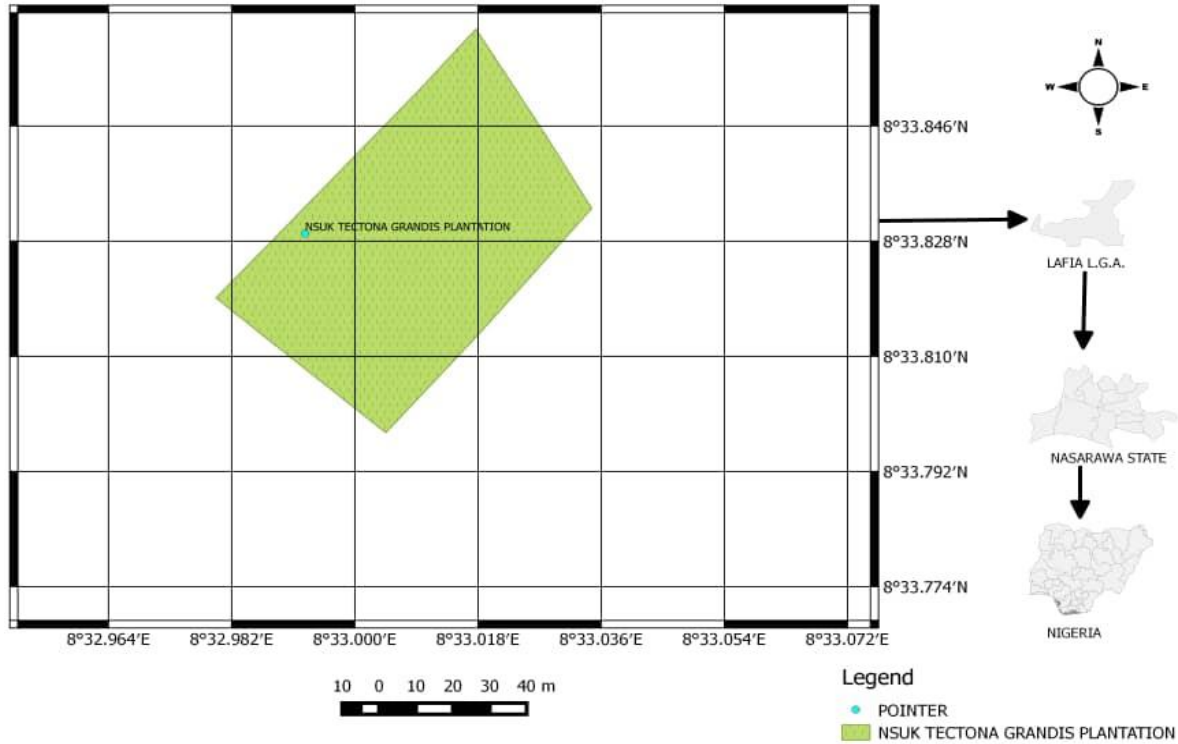


Fig. 1: Shows the Map of Nasarawa State University Forestry Teak Plantation

2.3. Data collection

Diameter at breast height (DBH) and heights of trees of *Tectona grandis* in each plot were measured. Also, coordinates of the study area was collected. Instruments and materials that were used during this research include; Haga altimeter, diameter tape, ground survey tape, ranging poles, lines, marker, tags and GPS.

2.3.1. Height-Diameter Models

$$H = 1.3 + \left(\frac{aDBH}{(b+DBH)}\right) + \epsilon \quad (1)$$

$$H = 1.3 + \left(\exp\left(\frac{a+b}{DBH}\right)\right) + \epsilon \quad (2)$$

$$H = 1.3 + \left(\exp\left(\frac{a+b}{DBH+1}\right)\right) + \epsilon \quad (3)$$

$$H = 1.3 + \left(\exp\left(\frac{a+b}{DBH}\right)^3\right) + \epsilon \quad (4)$$

2.4. Non-Linear Height-Diameter Mixed Effects Model

$$H = 1.3 + (a - be^{-cDBH}) + \epsilon \quad (3\text{-parameter asymptotic exponential) ---- Asymptotic} \quad (1)$$

$$H = 1.3 + \left(\frac{aDBH}{1+bDBH}\right) + \epsilon \quad \text{---- Michaelis menten} \quad (2)$$

Where:
1.3= height at Dbh, Dbh= Diameter at breast height; b₀ & b₁ model parameters;exp= exponential.

2.4.1. Model evaluation

The evaluation of the candidate models was based on graphical and numerical analysis of the data and model selection indices such as the Akaike information criterion (AIC) and Bayesian information criterion (BIC), Residual standard error (RSE), standard error (SE) and LOGLIK. They are mathematically expressed as follows:

2.4.2. AIC function

$$AIC = 2k - 2\ln(\hat{L})$$

Where
AIC= Akaike information criterion, K=Number of estimated parameters in the model
(\hat{L}) =Maximum value of the likelihood function for the model

2.4.3. BIC Function

$$BIC = -2 \cdot \ln p(x/k) \approx BIC = -2 \cdot \ln L + k \{ \ln(n) \}$$

Where

x= The observed data, n= The number of data points in x, the number of the observed or equivalently of the sample size, k= The number of free parameters to be estimated i.e the number of regression including intercept, p(x/k) = The probability of the observed data given by the parameter and L=The maximized value of the likelihood functions for the estimated model

2.4.4. Residual Standard Error

$$RSE = \sqrt{\frac{1}{n-2} RSS} = \sqrt{\frac{1}{n-2} \sum (y_i - \hat{y}_i)^2}$$

3. RESULTS

The results of height-diameter (H-D) model parameters and model selection indices of *Tectona grandis* is shown in Table 1. Based on the model selection indices, model two (2) has the lowest AIC (-1168.253), BIC (-1157.469) and RSE (0.02738), followed by model (4) with AIC (-1037.233), BIC (-1026.449) and RSE (0.03493), model one (1) with AIC (-975.4891), BIC (-964.7049) and RSE (0.03918), model three (3) with AIC (-845.8148), BIC (-835.0307) and RSE (0.04986) respectively. The probability levels of the model parameters were also highly significant at 0.05.

Table 1: Height-Diameter Model Selection Indices of *Tectona grandis*

| Models | Coefficients | AIC | BIC | RSE | Pr(> t) |
|--------|--------------------------------|-----------|-----------|---------|--------------------------|
| [1] | a = 8.9009227 b = 0.0452283 | -975.4891 | -964.7049 | 0.03918 | <2e-16 *** <2e-16 *** |
| [2] | a = 2.05700 b = -0.02504 | -1168.253 | -1157.469 | 0.02738 | <2e-16 *** <2e-16 *** |
| [3] | a = 1.059471 b = -0.025257 | -845.8148 | -835.0307 | 0.04986 | <2e-16 *** <2e-16 *** |
| [4] | a = 0.005685 b = 0.3477 | -1037.233 | -1026.449 | 0.03493 | <2e-16 *** <2e-16 *** |

*=significant, AIC=Akaike information criterion, BIC=Bayesian information criterion, RSE=root mean square error, Pr=p-value; a,b..=model parameters.

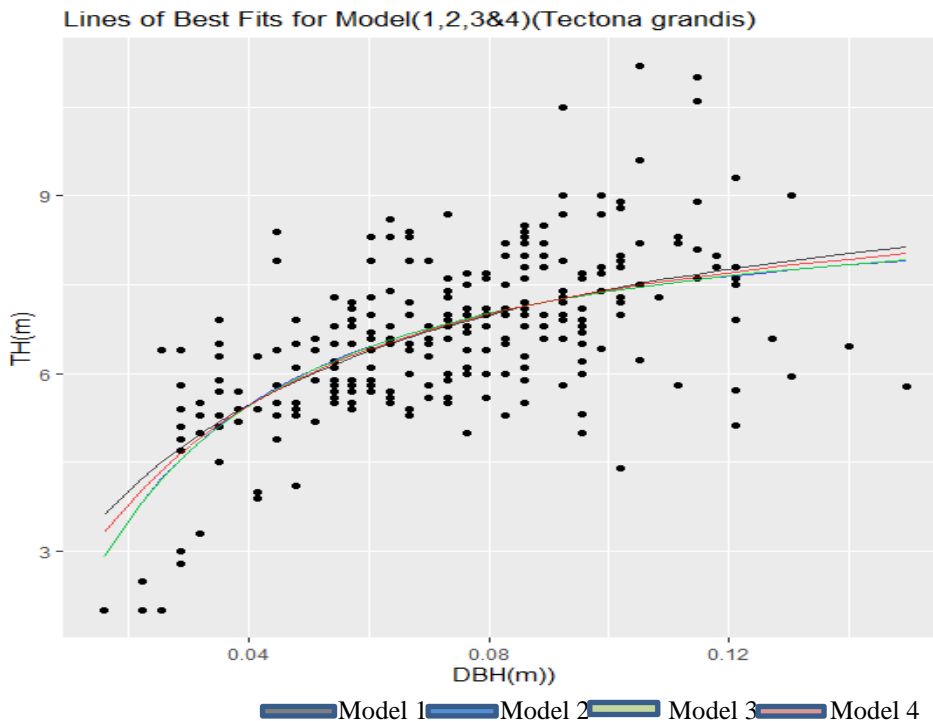


Figure 2: Line of Best Fit for the Four Height-Diameter Models

Table 2: Non-linear Mixed Effects Height-Diameter Model Selection Indices of *Tectona grandis* in the Study Area

| Fixed effects coefficients | | | | | | | | |
|-----------------------------|--|---|---|--|------------------|--|---------------|---------------|
| Mo dels | Coefficients | AIC | BIC | logLik | Residual | Pr(> t) | | |
| 1 | a = 8.197275 b= -9.040830 c= 3.658116 std Dev α = 0.7166316 β = 5.8771884 γ = 0.1434984 std Error δ_1 =0.446371 δ_2 =3.480789 δ_3 =0.144471 | 100.5421 | 116.9179 | -40.27104 | 0.609021 4 | 0.000 0 0.014 1 0.000 0 | | |
| 2 | a = 10.285240 b = 0.031708 std Dev α = 9.240240e-01 β =1.353063e-06 std Error δ_1 =0.9410362 δ_2 =0.0089093 | 95.27081 | 105.0963 | -41.6354 | 6.384774 e-01 | 0.000 0 0.001 2 | | |
| Random Effects coefficients | | | | | | | | |
| Mo dels | plots | a | B | c | AIC | BIC | logLik | Std Error |
| 1 | Plot C Plot A Plot B Plot D | 7.5761 61 7.3008 12 11.251 240 9.0820 02 | - 1251.78715 5 0.712948 -1.570585 -82.804758 | 4.7873 97 3.2551 88 2.7221 92 4.2956 53 | 100.5 421 | 116.9 179 | - 40.27104 | 0.628 4565 |
| 2 | Plot C Plot A Plot B Plot D | 9.3662 86 8.9750 41 19.236 09 10.004 349 2 | 0.02353730 0.03169165 0.12726169 0.01194367 | | 95.27 081 | 105.0 963 | -41.6354 | 0.624 4537 |

AIC=Akaike information criterion, BIC=Bayesian information criterion and a, b, c=model parameters.

Figure 2 illustrated the total heights of trees was plotted against diameter at breast heights. Also, five models were applied to estimate the lines of best fit for *Tectona grandis*. Model two (2) showed an increasing trend and has many data points on and around the line of best fit better than other four (4) regression models used.

The model selection indices of non-linear height-diameter mixed effects models with fixed and random effects is shown in Table 2. The result of fixed effects revealed that Michaelis Menten's model (model 2) had the lowest model selection indices of AIC (95.27081), BIC (105.0963), LOGLIK (-41.6354) and Residual (6.384774e⁻⁰¹), followed by Asymptotic model (model 1) with AIC (100.5421), BIC (116.9179), LOGLIK (-

40.27104) and Residual (0.6090214). The result of random effects models also showed that Michaelis Menten’s model had the lowest model selection indices of AIC (95.27081), BIC (105.0963), LOGLIK (-41.6354) and SE (0.6244537), followed by Asymptotic model

(model 1) with AIC (100.5421), BIC (116.9179), LOGLIK (-40.27104) and SE (0.6244537). The probability levels of the fixed and random effects parameters were also highly significant at 0.05.

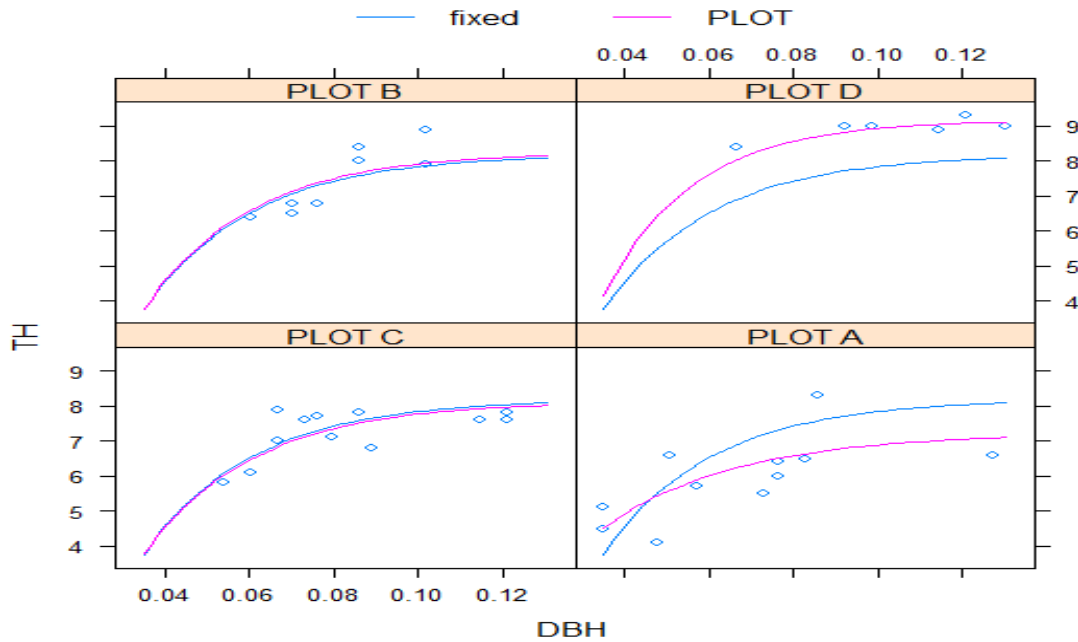


Figure 3: Fixed and random effects with asymptotic model

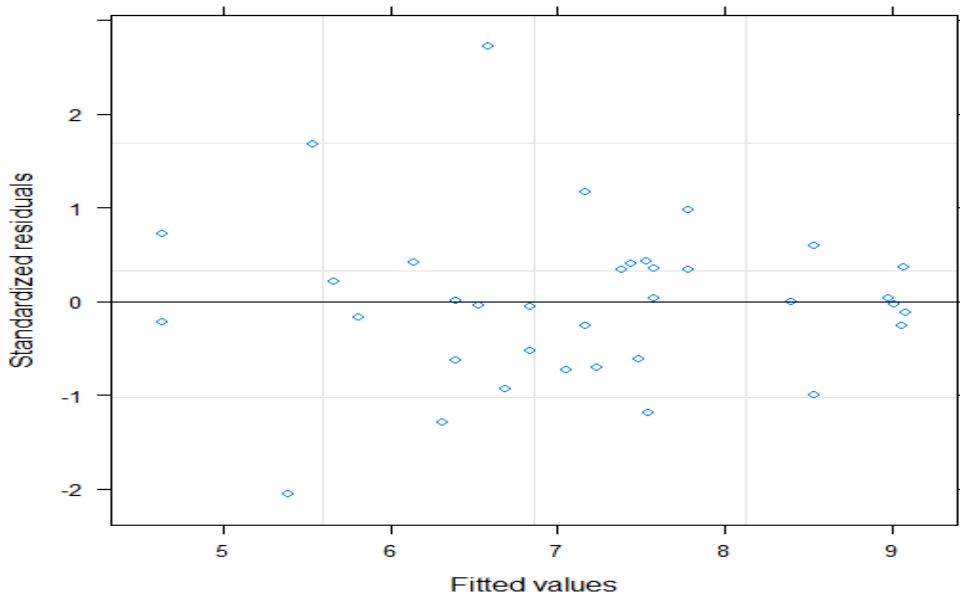


Figure 4: Standardized residuals plot against Fitted Values of Asymptotic Model with mixed effects

Asymptotic model was applied on diameter at breast heights to predict tree heights of *Tectona grandis* (Fig. 3) The Asymptotic model was applied to visualize the fixed and random effects of the model. The result unveiled that random effects produced a better line of best fit when compared with fixed effects line of best fit.

Standardized residuals was plotted against the fitted values of asymptotic model in order to test for normality of the *Tectona grandis* data set as illustrated in Figure 4. The result showed that the data points were evenly distributed around the mean line. Therefore, the data set was normally distributed.

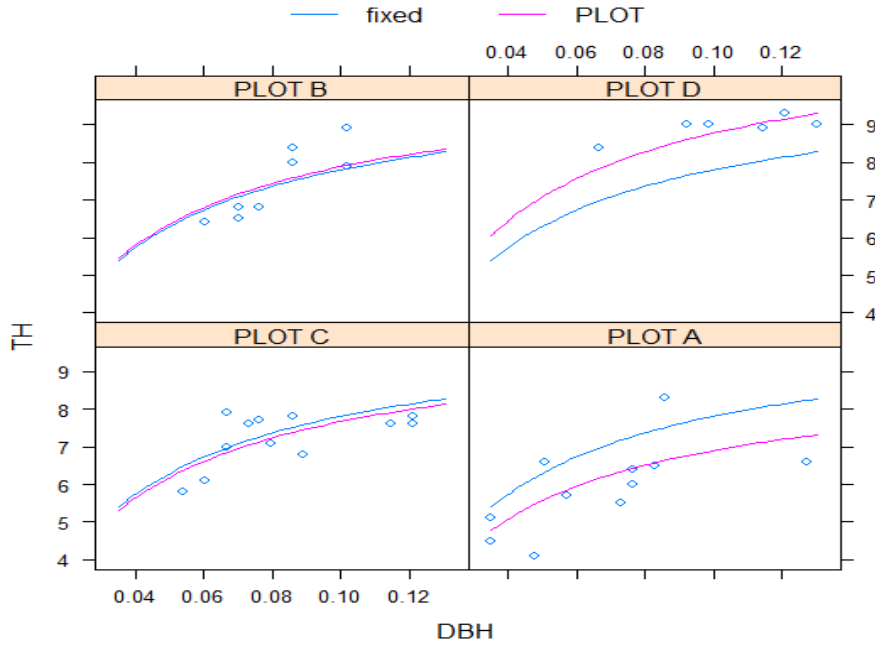


Figure 5: Fixed and Random Effects with Michaelis Menten Model

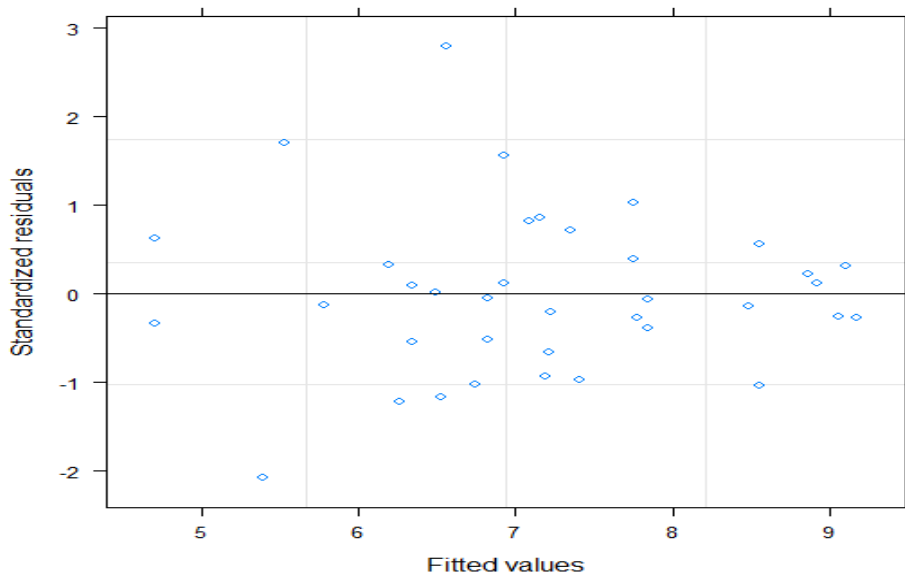


Figure.6: Standardized residuals plot against Fitted Values of Michaelis Menten Model with mixed effects

In Figure 5, Michaelis Menten’s model was applied on diameter at breast heights to predict tree heights of

Tectona grandis. The model was applied in order to visualize the fixed and random effects of the model. The

result unveiled that random effects produced a better line of best fit when compared with fixed effects line of best fit. It also showed an increasing trend and has many data points on and around the line of best fit better than fixed effects line of best fit.

In Figure 6, the Standardized residuals was plotted against the fitted values of Michaelis Menten's model to test for normality of the *Tectona grandis* data set. The result showed that the data set was evenly distributed around the mean line. Therefore, the data set was normally distributed.

4. DISCUSSION

Nonlinear Height-Diameter (H-D) modeling provides an easy access to estimating tree heights in a given plantation. Measurement of tree diameter at breast height is relatively simple, accurate and with low cost (Corral-Rivas et al., 2019). Conversely, tree height measurement is difficult, time consuming and expensive (Mehtätalo et al., 2015). It also unveils the contributions of tree growth attributes such as diameter at breast heights to tree total heights increment and serves as an integral parts to most statistical functions. Nonlinear H-D power models for the tree species group using the AIC and RSE indicate that the Nonlinear H-D models improved the accuracy of tree heights prediction (Mehtätalo et al., 2015).

The result of H-D model selection indices shows that model two had the lowest AIC (-1168.253), BIC (-1157.469), RSE (0.02738) with model parameters of a (2.05700), b(-0.02504) when compared to other three models applied. It implies that to every one unit increased in the tree heights, there was always a corresponding -0.02738 unit decreased in the tree diameters. Several nonlinear H-D models with single-variable (D) have been developed for different species (Ferraz-Filho et al., 2018). The model two aptly unveiled the nature of growth commonly observed in trees. As the trees increased in heights, their diameters decreased along the boles. It also indicated that the trees were still growing as observed in the slope of model two. In addition, the growing trend of the trees could be observed easily in the model two deterministic line of best fit. Therefore, model two (2) is the most suitable to predict tree heights of *Tectona grandis* in the study area.

It is spectacular to observe fixed effects estimates from either linear or non-linear modeling in most online articles. But the non-linear mixed effects height-diameter modeling provides an access to fixed and random effects estimates. Mixed-effects models simultaneously estimate both population-averaged (fixed effects) and subject-specific (random effects) parameters, allowing the variability for a given subject to be modeled (Pinheiro

and Bates, 2000). The result of non-linear mixed effects height-diameter model selection indices shows that Asymptotic and Michaelis Menten models were applied to predict *Tectona grandis* tree heights while comparing the fixed and random effect estimates in each model and examined the model selection indices. It was observed that the two parameter Michaelis Menten model had a lower estimates of model selection indices when compared to the 3-parameter asymptotic model. Also, the random effect estimates of the two parameter Michaelis Menten model had a better estimates when compared to the 3-parameter asymptotic models estimates.

However, the random effects estimates of Asymptotic and Michaelis Menten models was better than its fixed effects estimates. These models have been developed using either fixed-effect or mixed-effect techniques. In fixed-effect H-D models, the assumption of independence is violated (Ozcelik et al., 2018). The result of fixed and random effects with asymptotic model unveiled that asymptotic model was applied on diameter at breast heights to predict the tree heights. It was also applied in order to visualize the fixed and random effects of the model. The result revealed that random effects produced a better line of best fit when compared to fixed effects line of best fit of the same model. It also showed an increasing trend of deterministic line of best fit better than fixed effects line of best fit. The result of the fixed and random effects with Michaelis Menten Model unveiled that random effects produced a better line of best fit when compared with fixed effects line of best fit of Michaelis Menten model. The results are in line with Crecente-Campo et al., (2010) observed that mixed effect model are more accurate than the generalized ordinary least square height-diameter models. It also showed an increasing trend of deterministic line of best fit better than fixed effects line of best fit.

The result of Standardized residuals plotted against fitted values of asymptotic model with mixed effects showed that the data set was evenly distributed around the mean line. Therefore, the data is normally distributed. Also, the result of Standardized residuals plotted against Fitted Values of Michaelis Menten Model with mixed effects showed that the data was evenly distributed around the mean line. Therefore, the data set the sample plots was normally distributed of the within the study area.

5. CONCLUSION

- (i) The non-linear mixed-effect height-diameter modeling method unveiled both fixed and random effects estimates. It also provides an access to visualizing and comparing fixed and random effects estimates. The result of asymptotic and Michaelis Menten models on *Tectona grandis* to

predict its tree heights in the study area showed that non-linear mixed-effect height-diameter modeling method was better than generalized ordinary least square height-diameter modeling method. Though, they have their peculiar applications.

- (ii) The result of asymptotic and Michaelis Menten models also revealed that random effects produced a better within plots deterministic line of best fit when compared with fixed effects means line of best fit of the same model. Random effects estimations by non-linear mixed effect height-diameter modeling method is more robust and efficient to predict *Tectona grandis* tree heights than the generalized ordinary least square height-diameter modeling method (fixed effects modeling method) in the study area.

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